

# Light-front dynamic analysis of transition form factors in the process of $P \rightarrow V\ell\nu_\ell$

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## Abstract

We investigate the light-front zero-mode contribution to the weak transition form factors between pseudoscalar and vector mesons using a covariant fermion field theory model in  $(3+1)$  dimensions. In particular, we discuss the form factors  $a_-(q^2)$  and  $f(q^2)$  which have been suspected to have the zero-mode contribution in the  $q^+ = 0$  frame. While the zero-mode contribution in principle depends on the form of the vector meson vertex  $\Gamma^\mu = \gamma^\mu - (2k - P_V)^\mu/D$ , the form factor  $f(q^2)$  is found to be free from the zero mode if the denominator  $D$  contains the term proportional to the light-front longitudinal momentum fraction factor  $(1/x)^n$  of the struck quark with the power  $n > 0$ . Although the form factor  $a_-(q^2)$  is not free from the zero mode, the zero-mode contribution comes only either from the simple vertex  $\Gamma^\mu = \gamma^\mu$  term or from the other term just with a constant  $D$  (i.e.  $n = 0$ ), but not with the momentum-dependent denominator (i.e.  $D \sim (1/x)^n$  with  $n > 0$ ). We identify the zero-mode contribution to  $a_-(q^2)$  and incorporate it as a convolution of the zero-mode operator with the initial- and final-state light-front wave functions. The covariance (i.e. frame independence) of our model has been checked by performing the light-front calculations both in the  $q^+ = 0$  and  $q^+ > 0$  frames. We present our numerical result of the  $B \rightarrow \rho$

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transition for an explicit demonstration of our findings.

*Keywords:* Semileptonic decays; Weak form factors; Analytic continuation; Light-front zero mode

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## 1. Introduction

The exclusive semileptonic decay processes of heavy mesons generated a great excitement not only in extracting the most accurate values of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements but also in testing diverse theoretical approaches to describe the internal structure of hadrons. The great virtue of semileptonic decay processes is that the effects of the strong interaction can be separated from the effects of the weak interaction into a set of Lorentz-invariant form factors, i.e., the essential informations of the strongly interacting quark/gluon structure inside hadrons. Thus, the theoretical problem associated with analyzing semileptonic decay processes is essentially that of calculating the weak form factors.

Perhaps, one of the most well-suited formulations for the analysis of exclusive processes involving hadrons may be provided in the framework of light-front (LF) quantization [1]. For its simplicity and the predictive power of the hadronic form factors in low-lying ground-state hadrons, especially mesons, the LF constituent quark model (LFQM) based on the LF quantization has become a very useful and popular phenomenological tool to study various electroweak properties of mesons [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The simplicity on the LF quantization [1] is mainly attributed to the suppression of the vacuum fluctuations with the decoupling of complicated zero modes [13] and the conversion of the dynamical problem from boost to rotation. The suppression of vacuum fluctuations is due to the rational energy-momentum dispersion relation which correlates the signs of the LF energy  $k^- = k^0 - k^3$  and the LF longitudinal momentum  $k^+ = k^0 + k^3$ . However, the zero-mode complication in the matrix element has been noticed for the electroweak form factors involving a spin-1 particle [9, 10, 11, 14, 15]. A growing concern is to pin down which form factors receive the zero-mode contributions.

The main purpose of this work is to analyze the weak form factor  $a_-(q^2)$ , which has not been computed in our previous work of the semileptonic  $P \rightarrow V\ell\nu_\ell$  decays [15]. Unlike the form factors  $g(q^2)$ ,  $a_+(q^2)$ , and  $f(q^2)$ , which can be obtained from the plus component of the currents [15], one needs to use the perpendicular (or minus) components of the currents to obtain

$a_-(q^2)$ . In this work, we use the perpendicular components of the axial-vector currents with the transverse polarization to obtain  $a_-(q^2)$  and analyze the existence/absence of the zero mode.

The paper is organized as follows. In Section 2, we discuss the  $P \rightarrow V\ell\nu_\ell$  semileptonic decays using an exactly solvable model based on the covariant Bethe-Salpeter (BS) model of  $(3+1)$ -dimensional fermion field theory. In Section 3, we present our LF calculation of the weak form factors in the  $q^+ > 0$  frame and discuss the result in the  $q^+ \rightarrow 0$  limit for the analysis on the existence/absence of the zero-mode contribution to the form factors. Especially, we identify the zero-mode contribution to  $a_-(q^2)$  and incorporate it as a convolution of the zero-mode operator with the initial- and final-state LF wave functions. We also present our numerical result for the explicit demonstration of our findings. Summary and discussion follow in Section 4. In the appendices A and B, we summarize the LF results of the trace terms for the weak current matrix element and the results of the weak form factors obtained from the  $q^+ > 0$  frame, respectively.

## 2. Model Description

The Lorentz-invariant transition form factors  $g$ ,  $f$ ,  $a_+$ , and  $a_-$  between a pseudoscalar meson with four-momentum  $P_1$  and a vector meson with four-momentum  $P_2$  and helicity  $h$  are defined by the matrix elements of the electroweak current  $J_{V-A}^\mu$  from the initial-state  $|P_1; 00\rangle$  to the final-state  $|P_2; 1h\rangle$  [16]:

$$\begin{aligned} \langle P_2; 1h | J_{V-A}^\mu | P_1; 00 \rangle &= ig(q^2) \varepsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* P_\alpha q_\beta - f(q^2) \epsilon^{*\mu} \\ &\quad - a_+(q^2) (\epsilon^* \cdot P) P^\mu - a_-(q^2) (\epsilon^* \cdot P) q^\mu, \end{aligned} \quad (1)$$

where  $P = P_1 + P_2$  and  $q = P_1 - P_2$  is the four-momentum transfer to the lepton pair ( $\ell\nu_\ell$ ). The polarization vector  $\epsilon^* = \epsilon^*(P_2, h)$  of the final-state vector meson satisfies the Lorentz condition  $\epsilon^* \cdot P_2 = 0$ . The polarization vectors used in this analysis are given by

$$\begin{aligned} \epsilon^\mu(\pm 1) &= [\epsilon^+, \epsilon^-, \epsilon_\perp] = \left[ 0, \frac{2}{P_2^+} \epsilon_\perp(\pm) \cdot \mathbf{P}_{2\perp}, \epsilon_\perp(\pm 1) \right], \\ \epsilon_\perp(\pm 1) &= \mp \frac{(1, \pm i)}{\sqrt{2}}, \quad \epsilon^\mu(0) = \frac{1}{M_2} \left[ P_2^+, \frac{\mathbf{P}_{2\perp}^2 - M_2^2}{P_2^+}, \mathbf{P}_{2\perp} \right]. \end{aligned} \quad (2)$$

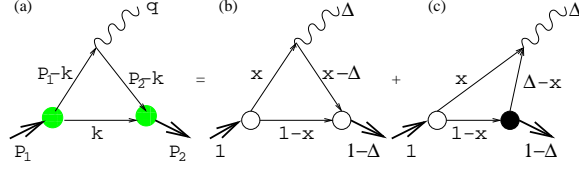


Figure 1: The covariant diagram (a) corresponds to the sum of the LF valence diagram (b) and the nonvalence diagram (c). The large white and black blobs at the meson-quark vertices in (b) and (c) represent the ordinary LF wave function and the nonvalence wave function vertices, respectively.

While the form factor  $g(q^2)$  is associated with the vector current  $V^\mu$ , the rest of the form factors  $f(q^2)$ ,  $a_+(q^2)$ , and  $a_-(q^2)$  are coming from the axial-vector current  $A^\mu$ .

The transition form factors defined in Eq. (1) are often given by the Bauer, Stech, and Wirbel (BSW) convention [17],

$$\begin{aligned}
V(q^2) &= -(M_1 + M_2)g(q^2), \\
A_1(q^2) &= -\frac{f(q^2)}{M_1 + M_2}, \\
A_2(q^2) &= (M_1 + M_2)a_+(q^2), \\
A_0(q^2) &= \frac{-1}{2M_2} \left[ f(q^2) + q \cdot P a_+(q^2) + q^2 a_-(q^2) \right], \quad (3)
\end{aligned}$$

where  $M_1$  and  $M_2$  are the physical pseudoscalar and vector meson masses, respectively, and  $q \cdot P = M_1^2 - M_2^2$ .

The exactly solvable model based on the covariant BS model of  $(3+1)$ -dimensional fermion field theory [8, 14] enables us to derive the transition form factors between pseudoscalar and vector mesons explicitly. The covariant diagram shown in Fig. 1(a) is in general equivalent to the sum of the LF valence diagram [Fig. 1(b)] and the nonvalence diagram [Fig. 1(c)]. The matrix element  $\langle J_{V-A}^\mu \rangle_h \equiv \langle P_2; 1h | J_{V-A}^\mu | P_1; 00 \rangle$  obtained from the covariant diagram of Fig. 1(a) is given by

$$\langle J_{V-A}^\mu \rangle_h = ig_1 g_2 \Lambda_1^2 \Lambda_2^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(S_{V-A}^\mu)_h}{N_{\Lambda_1} N_1 N_{\bar{q}} N_2 N_{\Lambda_2}}, \quad (4)$$

where  $g_{1(2)}$  is the normalization factor which can be fixed by requiring charge form factor of pseudoscalar (vector) meson to be unity at  $q^2 = 0$ . To regularize the covariant fermion triangle loop in  $(3+1)$  dimensions, we replace the

point gauge-boson vertex  $\gamma^\mu(1 - \gamma_5)$  by a non-local (smeared) gauge-boson vertex  $(\Lambda_1^2/N_{\Lambda_1})\gamma^\mu(1 - \gamma_5)(\Lambda_2^2/N_{\Lambda_2})$ , where  $N_{\Lambda_{1(2)}} = p_{1(2)}^2 - \Lambda_{1(2)}^2 + i\epsilon$ , and  $\Lambda_1$  and  $\Lambda_2$  play the role of momentum cut-offs similar to the Pauli-Villars regularization. The rest of the denominators in Eq. (4) coming from the intermediate fermion propagators in Fig. 1(a) are given by

$$N_1 = p_1^2 - m_1^2 + i\epsilon, \quad N_{\bar{q}} = k^2 - m^2 + i\epsilon, \quad N_2 = p_2^2 - m_2^2 + i\epsilon, \quad (5)$$

where  $m_{1(2)}$  and  $m$  are the masses of the constituents carrying the intermediate four-momenta  $p_{1(2)} = P_{1(2)} - k$  and  $k$ , respectively.

The trace term  $(S_{V-A}^\mu)_h$  in Eq. (4) is given by

$$(S_{V-A}^\mu)_h = \text{Tr}[(\not{p}_2 + m_2)\gamma^\mu(1 - \gamma_5)(\not{p}_1 + m_1)\gamma_5(-\not{k} + m)\epsilon^* \cdot \Gamma], \quad (6)$$

where the initial-state pseudoscalar meson vertex operator is  $\gamma^5$  and the final-state vector meson vertex operator  $\Gamma^\mu$  is given by

$$\Gamma^\mu = \gamma^\mu - \frac{(P_2 - 2k)^\mu}{D}. \quad (7)$$

While  $\gamma^\mu$  is intrinsic to the vector meson vertex, the model-dependence of vector meson is implemented through the factor  $D$  in Eq. (7). Frequently used  $D$  factor is either constant ( $D_{\text{con}}$ ) or covariant ( $D_{\text{cov}}$ ):

$$\begin{aligned} (1) \quad D_{\text{con}} &= M_2 + m_2 + m, \\ (2) \quad D_{\text{cov}} &= \frac{2k \cdot P_2 + M_2(m_2 + m) - i\epsilon}{M_2}. \end{aligned} \quad (8)$$

We note that the  $D$  factor behaves like  $(1/x)^n$  as the LF longitudinal momentum fraction  $x$  goes to zero (i.e.  $x \rightarrow 0$ ), where  $n = 0$  and  $1$  for the cases of (1) and (2) of Eq. (8), respectively.

As discussed in our previous work [15], Jaus's prescription [10] to find the zero mode is limited to the case of  $D = D_{\text{con}}$ . In this work, we analyze  $a_-(q^2)$  for both cases of (1) and (2) of Eq. (8) and confirm again that Jaus's prescription applies only to the case (1) but not to the case (2). We also apply the LF version of the  $D$  factor, i.e.  $D_{\text{LF}} = M_0 + m_2 + m$  with the invariant mass  $M_0$  of the vector meson. This case corresponds to  $n = 1/2$  and Jaus's prescription doesn't apply to this case either as we will discuss in the next section.

### 3. Light-front calculation of the weak form factors

In the  $q^+ > 0$  frame, the covariant diagram Fig. 1(a) corresponds to the sum of the LF valence diagram (b) defined in  $0 < k^+ < P_2^+$  region and the nonvalence diagram (c) defined in  $P_2^+ < k^+ < P_1^+$  region. The large white and black blobs at the meson-quark vertices in (b) and (c) represent the ordinary LF wave functions and the non-wave-function vertex [14], respectively.

Defining  $\Delta = q^+/P_1^+$  and the longitudinal momentum fraction factor  $x = p_1^+/P_1^+$  ( $1 - x = k^+/P_1^+$ ) for the struck (spectator) quark, we should note that the nonvalence region (i.e.  $0 < x < \Delta$ ) of integration shrinks to the end point  $x = 0$  in the  $q^+ \rightarrow 0$  (i.e.  $\Delta \rightarrow 0$ ) limit. The virtue of taking  $q^+ = 0$  frame is to obtain the form factor by calculating only the valence diagram (i.e.  $0 < x < 1$ ) because the nonvalence diagram does not contribute if the integrand is free from the singularity in  $p_1^- \sim 1/x$ . However, if the integrand has a singularity as  $x \rightarrow 0$ , then one should take into account not only the valence diagram but also the nonvalence diagram because the latter can also give nonvanishing contribution even if the integration range of this diagram shrinks to the end point  $x = 0$ . Thus, one needs to analyze carefully if the contribution from the nonvalence diagram in the  $q^+ = 0$  frame occurs or not, in order to correctly utilize the  $q^+ = 0$  frame without any error. Calling such contributions from the end point  $x = 0$  as zero modes, we investigate them for the form factors in the  $P \rightarrow V\ell\nu_\ell$  transition.

In order to check the existence/absence of the zero-mode contribution to the hadronic matrix element given by Eq. (4), we first choose  $q^+ > 0$  frame and then take  $q^+ \rightarrow 0$  limit. Our analysis for the zero mode is based on the  $q^+ = 0$  [or Drell-Yan(DY)] frame [18]:

$$\begin{aligned} P_1 &= (P_1^+, P_1^-, \mathbf{P}_{1\perp}) = (P_1^+, \frac{M_1^2}{P_1^+}, \mathbf{0}_\perp), \quad P_2 = (P_1^+, \frac{M_2^2 - q^2}{P_1^+}, -\mathbf{q}_\perp), \\ q &= (0, \frac{M_1^2 - M_2^2 + q^2}{P_1^+}, \mathbf{q}_\perp), \end{aligned} \quad (9)$$

where  $q^2 = -\mathbf{q}_\perp^2$  is the spacelike gauge boson momentum transfer. The weak form factors in the timelike  $q^2$  region can be obtained by the analytic continuation from the spacelike  $q^2$  region.

The relations between the current matrix elements and the weak form

factors in this  $q^+ = 0$  frame are as follows:

$$\begin{aligned}
g^{\text{DY}}(q^2) &= -\frac{\sqrt{2}q^R}{q^2}\langle J_V^+ \rangle_{h=1}, \quad a_+^{\text{DY}}(q^2) = -\frac{q^R}{q^2\sqrt{2}}\langle J_A^+ \rangle_{h=1}, \\
f^{\text{DY}}(q^2) &= (q^2 - q \cdot P)a_+^{\text{DY}}(q^2) + M_2\langle J_A^+ \rangle_{h=0}, \\
a_-^{\text{DY}}(q^2) &= a_+^{\text{DY}}(q^2) - \frac{1}{q^2}[f^{\text{DY}}(q^2) + \frac{\sqrt{2}}{q^L}\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}], \quad (10)
\end{aligned}$$

where  $q^{R(L)} = q_x \pm iq_y$ . Since the form factors ( $g, a_+, f$ ) have been analyzed in our previous work [15], we focus on the calculation of the form factor  $a_-^{\text{DY}}(q^2)$  (i.e.  $\langle J_A^\perp \rangle_{h=1}$ ) to find if the zero mode exists or not in this work.

### 3.1. Valence contribution

In the valence region  $\Delta < x < 1$ , the pole  $k^- = k_{\text{on}}^- = (\mathbf{k}_\perp^2 + m_{\bar{q}}^2 - i\epsilon)/k^+$  (i.e., the spectator quark) is located in the lower half plane of the complex  $k^-$ -variable. Thus, the Cauchy integration formula for the  $k^-$  integral in Eq. (4) gives

$$\langle J_{V(A)}^\mu \rangle_h = \frac{N}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi_1(x, \mathbf{k}_\perp) [S_{V(A)}^\mu]_h \chi_2(x, \mathbf{k}'_\perp), \quad (11)$$

where  $N = g_1 g_2 \Lambda_1^2 \Lambda_2^2$ . The LF vertex functions  $\chi_1$  and  $\chi_2$  are given by

$$\chi_{1(2)}(x, \mathbf{k}_\perp^{(\prime)}) = \frac{1}{x^2(M_{1(2)}^2 - M_0^{(\prime)2})(M_{1(2)}^2 - M_{\Lambda_{1(2)}}^{(\prime)2})}, \quad (12)$$

where  $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$  and

$$M_0^{(\prime)2} = \frac{\mathbf{k}_\perp^{(\prime)2} + m^2}{1-x} + \frac{\mathbf{k}_\perp^{(\prime)2} + m_{1(2)}^2}{x}, \quad M_{\Lambda_{1(2)}}^{(\prime)2} = M_0^{(\prime)2}(m_{1(2)} \rightarrow \Lambda_{1(2)}). \quad (13)$$

In our trace term  $[S_{V(A)}^\mu]_h$  calculation, we separate Eq. (6) into the on-mass-shell propagating part  $[S_{V(A)}^\mu]_h^{\text{on}}$  and the off-mass-shell instantaneous part  $[S_{V(A)}^\mu]_h^{\text{inst}}$ , i.e.

$$[S_{V(A)}^\mu]_h = [S_{V(A)}^\mu]_h^{\text{on}} + [S_{V(A)}^\mu]_h^{\text{inst}}, \quad (14)$$

via

$$\not{p} + m = (\not{p}_{\text{on}} + m) + \frac{1}{2}\gamma^+(p^- - p_{\text{on}}^-). \quad (15)$$

While the on-mass-shell part indicates that all three quarks are on their respective mass shell, i.e.  $k^- = k_{\text{on}}^-$  and  $p_i^- = p_{i\text{on}}^- (i = 1, 2)$ , the instantaneous part includes the term proportional to  $\delta p_i^- = p_i^- - p_{i\text{on}}^- (i = 1, 2)$  and  $\delta k^- = (k^- - k_{\text{on}}^-)$  [14]. The explicit forms of  $[S_{V(A)}^\mu]_h$  in Eq. (6) are presented in the appendix A.

### 3.2. Zero-mode contribution

In the nonvalence region  $0 < x < \Delta$ , the poles are at  $p_1^- = p_{1\text{on}}^-(m_1) = (m_1^2 + \mathbf{k}_\perp^2 - i\epsilon)/p_1^+$  (from the struck quark propagator) and  $p_1^- = p_{1\text{on}}^-(\Lambda_1) = (\Lambda_1^2 + \mathbf{k}_\perp^2 - i\epsilon)/p_1^+$  (from the smeared quark-gauge-boson vertex), which are located in the upper half plane of the complex  $k^-$ -variable. To investigate the zero-mode contribution, we need to analyze the nonvalence diagram [Fig. 1(c)] in the  $\Delta \rightarrow 0$  limit, where the nonvalence region shrinks to the end point  $x = 0$ .

To handle the complexity of treating double  $p_1^-$ -poles from  $N_{\Lambda_1}$  and  $N_1$ , we decompose the product of five denominators given in Eq. (4) into a sum of terms containing three propagators as follows:

$$\frac{1}{N_{\Lambda_1} N_1 N_{\bar{q}} N_2 N_{\Lambda_2}} = \frac{1}{(\Lambda_1^2 - m_1^2)(\Lambda_2^2 - m_2^2)} \frac{1}{N_{\bar{q}}} \left( \frac{1}{N_{\Lambda_1}} - \frac{1}{N_1} \right) \left( \frac{1}{N_{\Lambda_2}} - \frac{1}{N_2} \right). \quad (16)$$

From this decomposition, one may have zero-mode contribution proportional to  $\delta(x)$  from the  $p_1^-$ -pole<sup>1</sup> (if exists) in the numerator. For instance, the  $k^-$  integration of  $p_1^-/N_{\bar{q}}N_{\Lambda_1}N_{\Lambda_2}$  having  $p_1^- = p_{1\text{on}}^-(\Lambda_1)$  pole (i.e.  $N_{\Lambda_1} \rightarrow 0$ ) gives the following nonvanishing zero-mode contribution

$$\lim_{\Delta \rightarrow 0} \int_{nv} dk^- \frac{p_1^-}{N_{\bar{q}} N_{\Lambda_1} N_{\Lambda_2}} = 2\pi i \frac{\delta(x)}{\Lambda_{2\perp}^2 - \Lambda_{1\perp}^2} \ln \frac{\Lambda_{2\perp}^2}{\Lambda_{1\perp}^2}, \quad (17)$$

where  $\Lambda_{i\perp}^2 = \Lambda_i^2 + \mathbf{p}_{i\perp}^2$ . The appearance of  $\delta(x)$  in our analysis is closely related to the findings in Ref. [13]. It is very important to note that such zero mode in Eq. (17) is absent if  $p_1^-$  is combined with a factor  $x^n$  with  $n > 0$ , i.e.  $x^n p_1^-/(N_{\bar{q}}N_{\Lambda_1}N_{\Lambda_2})$ . From the power counting of  $x$  for the  $D$  factor used in the present analysis, one can easily see that the nonvanishing zero-mode contribution to  $(p_1^-/D)/(N_{\bar{q}}N_{\Lambda_1}N_{\Lambda_2})$  exists only when  $D = D_{\text{con}}$  (i.e.  $n = 0$ ),

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<sup>1</sup>Note that  $p_2^-$  and  $-k^-$  show the same singular behavior as  $p_1^-$ , i.e.  $p_1^- (= p_2^- = -k^-) \sim 1/x$  as  $x \rightarrow 0$ .



but absent when  $D_{\text{cov}}$  (i.e.  $n = 1$ ) or  $D_{\text{LF}} = M'_0 + m_2 + m$  (i.e.  $n = 1/2$ ) is used.

In our previous work [15] for the calculation of the form factors  $g(q^2)$ ,  $a_+(q^2)$ , and  $f(q^2)$ , we found that only the form factor  $f(q^2)$  (i.e.  $\langle J_A^+ \rangle_{h=0}$ ) may receive the zero-mode contribution from the  $p_1^-$  term in  $(S_A^+)_{h=0}$ . From the power counting rule for  $p_1^-$  (or  $1/x$ ) in  $(S_A^+)_{h=0}$ , we obtained the suspected zero-mode contribution as  $(S_A^+)_{h=0}^{\text{Z.M.}} = \lim_{\Delta \rightarrow 0} (S_A^+)_{h=0}^{\text{inst}} = 2\epsilon_{h=0}^{*+}(p_1^-/D)$ . This contribution is nonvanishing only if  $D = D_{\text{con}}$  but vanishes if  $\Gamma^\mu = \gamma^\mu$  (i.e.  $1/D = 0$ ),  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$  is used [15].

To find the zero-mode contribution to  $a_-(q^2)$  defined by Eq. (10), we need to analyze the zero-mode contribution to  $\langle J_A^\perp \rangle_{h=1}$  since it may come from the  $p_1^-$  term in  $(S_A^\perp)_{h=1}$ . From the power counting rule for  $p_1^-$  in  $(S_A^\perp)_{h=1}$ , we find

$$(S_A^\perp)_{h=1}^{\text{Z.M.}} = \lim_{\Delta \rightarrow 0} (S_A^\perp)_{h=1}^{\text{inst}} = 2p_1^- \left[ (m_1 + m_2)\epsilon_\perp^* + 2\frac{k_{\text{on}} \cdot \epsilon^*}{D}(2\mathbf{p}_{1\perp} - \mathbf{q}_\perp) \right]. \quad (18)$$

We note that only the instantaneous part  $(S_A^\perp)_{h=1}^{\text{inst}}$  in Eq. (A.4) contributes to the zero mode. The first term in the square bracket of Eq. (18) comes from the model-independent intrinsic  $\gamma^\mu$  part, while the second term comes from the  $(2k - P_V)^\mu/D$  part. It is important to note that the zero-mode contribution to  $(S_A^\perp)_{h=1}$  comes already from the model-independent intrinsic  $\gamma^\mu$  part in the computation of  $a_-(q^2)$ . As in the case of  $f(q^2)$ , however, the zero-mode contribution from the  $p_1^-/D$  term is nonvanishing only if  $D = D_{\text{con}}$  but vanishes if  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$ .

The net zero-mode contribution to  $\langle J_A^\perp \rangle_{h=1}$  is then obtained as  $\langle J_A^\perp \rangle_{h=1}^{\text{Z.M.}} = [J_{\Lambda_1\Lambda_2}^\perp]_{\text{Z.M.}} - [J_{\Lambda_1 m_2}^\perp]_{\text{Z.M.}} - [J_{m_1\Lambda_2}^\perp]_{\text{Z.M.}} + [J_{m_1 m_2}^\perp]_{\text{Z.M.}}$  from the decomposition of the denominators according to Eq. (16). For instance, we define the zero mode contribution to  $1/N_{\bar{q}}N_{\Lambda_1}N_{\Lambda_2}$  term in Eq. (16) as

$$[J_{\Lambda_1\Lambda_2}^\perp]_{\text{Z.M.}} = \lim_{\Delta \rightarrow 0} \int_{nv} \frac{d^4 k}{(2\pi)^4} \frac{[S_A^\perp(p_1^- = p_{1\text{on}}(\Lambda_1))]_{h=1}^{\text{Z.M.}}}{N_{\bar{q}}N_{\Lambda_1}N_{\Lambda_2}}. \quad (19)$$

The zero-mode contributions to the other three terms can be defined the same way as in Eq. (19). Therefore, as far as the  $D_{\text{LF}}$  or  $D_{\text{cov}}$  is used, the nonvanishing zero-mode contribution to  $(S_A^\perp)_{h=1}^{\text{Z.M.}}$  comes only from the intrinsic  $\gamma^\mu$  part. In this case, the nonvanishing zero-mode contribution to  $\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}$  is given by

$$\begin{aligned} \langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}^{\text{Z.M.}} &= \frac{N}{8\pi^2(\Lambda_1^2 - m_1^2)(\Lambda_2^2 - m_2^2)} \frac{q^L}{\sqrt{2}} (m_1 + m_2) \\ &\times \int_0^1 dz \ln \left( \frac{B_{\Lambda_1 m_2} B_{m_1 \Lambda_2}}{B_{\Lambda_1 \Lambda_2} B_{m_1 m_2}} \right), \end{aligned} \quad (20)$$

where

$$B_{ab} = (1 - z)a^2 + zb^2 - z(1 - z)q^2. \quad (21)$$

### 3.3. Effective inclusion of the zero-mode in the valence region

We may identify the zero-mode operator that is convoluted with the initial and final state valence wave functions to generate the zero-mode contribution given by Eq. (20). Since our findings agree with Jaus's results for the intrinsic  $\gamma^\mu$  part as well as the model-dependent  $(P_2 - 2k)^\mu/D$  part if the factor  $D$  is constant, our method for those parts that we agree with Jaus can also be realized effectively by the Jaus's method [10] using the orientation of the LF plane characterized by the invariant equation  $\omega \cdot x = 0$  [19, 20], where  $\omega$  is an arbitrary lightlike four vector. While the physical amplitudes should not depend on the orientation of the LF plane, the LF matrix elements can acquire a spurious  $\omega$  dependence. This problem is closely associated with the violation of rotational invariance in the computation of the matrix element of a one-body current. In order to treat the complete Lorentz structure of a hadronic matrix element, the authors in [10, 19] have developed a method to identify and separate spurious ( $\omega$  dependent) contributions to the hadronic form factors. Below, we summarize the result of zero-mode contribution obtained from the method by Jaus [10] and discuss the equivalence to our result of zero-mode contribution.

By adopting the  $\omega$  dependent LF covariant approach as in [10, 19], we identify the zero-mode operator that is convoluted with the initial and final state valence wave functions to generate the zero-mode contribution to the form factor  $a_-(q^2)$ . In order to do this, we first decompose the four vector  $p_1^\mu$  in terms of  $P, q$ , and  $\omega$  with  $\omega = (1, 0, 0, -1)$  as follows [10]:

$$p_1^\mu = P^\mu A_1^{(1)} + q^\mu A_2^{(1)} + \frac{1}{\omega \cdot P} \omega^\mu C_1^{(1)}. \quad (22)$$

The coefficients in Eq. (22) are given by

$$\begin{aligned}
A_1^{(1)} &= \frac{\omega \cdot p_1}{\omega \cdot P} = \frac{x}{2}, \\
A_2^{(1)} &= \frac{1}{q^2} \left( p_1 \cdot q - (q \cdot P) \frac{\omega \cdot p_1}{\omega \cdot P} \right) = \frac{x}{2} + \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2}, \\
C_1^{(1)} &= p_1 \cdot P - P^2 A_1^{(1)} - q \cdot P A_2^{(1)} = Z_2 - N_{\bar{q}},
\end{aligned} \tag{23}$$

where

$$Z_2 = x(M_1^2 - M_0^2) + m_1^2 - m_{\bar{q}}^2 + (1 - 2x)M_1^2 - [q^2 + q \cdot P] \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2}. \tag{24}$$

Note that only the coefficient  $C_1^{(1)}$  which is combined with  $\omega^\mu$  depends on  $p_1^-$  (i.e. zero mode). In this exactly solvable BS model, the zero-mode contribution from  $p_1^-$  is exactly opposite to that from  $N_{\bar{q}}$  [21], i.e.

$$\begin{aligned}
I[p_1^-]_{\text{Z.M.}} &= iN \int_{\text{Z.M.}} \frac{d^4 k}{(2\pi)^4} \frac{p_1^-}{N_{\Lambda_1} N_1 N_{\bar{q}} N_2 N_{\Lambda_2}} \\
&= \frac{N}{16\pi^2 (\Lambda_1^2 - m_1^2) (\Lambda_2^2 - m_2^2)} \int_0^1 dz \ln \left( \frac{B_{\Lambda_1 m_2} B_{m_1 \Lambda_2}}{B_{\Lambda_1 \Lambda_2} B_{m_1 m_2}} \right) \\
&= -I[N_{\bar{q}}]_{\text{Z.M.}}.
\end{aligned} \tag{25}$$

Furthermore, the zero-mode contribution  $I[N_{\bar{q}}]_{\text{Z.M.}}$  from  $N_{\bar{q}}$  is exactly the same as the valence contribution  $I[Z_2]_{\text{val}}$  from  $Z_2$ , where  $I[Z_2]_{\text{val}}$  is given by

$$I[Z_2]_{\text{val}} = \frac{N}{16\pi^3} \int_0^1 \frac{dx}{1-x} \int d^2 \mathbf{k}_\perp \chi_1(x, \mathbf{k}_\perp) \chi_2(x, \mathbf{k}'_\perp) Z_2. \tag{26}$$

From the identities in Eqs. (25) and (26), the replacement  $N_{\bar{q}} \rightarrow Z_2$  (or equivalently  $p_1^- \rightarrow -Z_2$ ) in the spurious  $\omega$  dependent (i.e. the zero-mode related) term  $C_1^{(1)}$  in Eq. (23) makes the amplitude free of any  $\omega$  dependence, and effectively includes the zero-mode contribution from  $p_1^-$  in the valence region with the help of Eq. (26). Using this prescription, we can effectively include the zero-mode contribution to  $\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}$  in the LF valence region. For the intrinsic  $\gamma^\mu$  part, as an example, the nonvanishing zero-mode contribution to  $\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}$  is obtained as

$$\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}^{\text{Z.M.}} = -\frac{q^L}{\sqrt{2}} \frac{N}{8\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 \mathbf{k}_\perp \chi_1 \chi_2 (m_1 + m_2) Z_2, \tag{27}$$

and the full result for  $\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}$  is given by  $\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}^{\text{full}} = \langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}^{\text{val}} + \langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}^{\text{Z.M.}}$ . We should note that Eq. (20) and Eq. (27) coincide exactly.

### 3.4. Transition Form Factors for $D = D_{\text{cov}}$ and $D_{\text{LF}}$

Since more realistic LFQM uses  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$  instead of  $D_{\text{con}}$ , we obtain the transition form factors which are valid when  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$ . In this case, the three form factors  $g(q^2)$ ,  $a_+(q^2)$ , and  $f(q^2)$  can be obtained without encountering the zero-mode contribution as we have already proved in Ref. [15]. On the other hand, the form factor  $a_-(q^2)$  receives the zero-mode contribution from  $\langle J_A^\perp \cdot \mathbf{q}_\perp \rangle_{h=1}^{\text{Z.M.}}$ , which comes from the intrinsic  $\gamma^\mu$  part but not from the model-dependent part with  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$  factor as we discussed in the previous section.

Since the frame-independent (or covariant) form factors ( $g^{\text{DY}}, a_+^{\text{DY}}, f^{\text{DY}}$ ) have already been given in our previous analysis [15], we do not list them here again. Including the zero-mode contribution given by Eq. (27), we now obtain the form factor  $a_-^{\text{DY}}(q^2)$  as follows:

$$\begin{aligned}
a_-^{\text{DY}}(q^2) = & \frac{N}{8\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2\mathbf{k}_\perp \chi_1 \chi_2 \left\{ (2x-3)\mathcal{A}_1 + \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2} [(7-6x)m_1 \right. \\
& - m_2 - (4-6x)m] + \frac{2}{q^2} (m_1 - m) \left[ xZ_2 - 2\frac{(\mathbf{k}_\perp \cdot \mathbf{q}_\perp)^2}{q^2} \right] \\
& - \frac{2}{(1-x)D} \left( (\mathbf{k}_\perp \cdot \mathbf{k}'_\perp + \mathcal{A}_1\mathcal{B}_2) \left[ (1-x) + \frac{Z_2}{q^2} - \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2} \right] \right. \\
& - \left[ (1-x)Z_2 + 2\mathbf{k}_\perp^2 + 2m\mathcal{A}_1 - 2(1-x)[M_2^2 - q^2 \right. \\
& \left. \left. \left. + (m_2 + m)(m_1 - m)] \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2} \right] \left[ (1-x) - \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{q^2} \right] \right) \right\}, \quad (28)
\end{aligned}$$

where  $\mathcal{A}_i = (1-x)m_i + xm$  ( $i = 1, 2$ ) and  $\mathcal{B}_2 = xm - (1-x)m_2$ .

In Table 1, we summarize our findings on the existence/absence of the zero-mode contribution to the hadronic form factors ( $g, a_\pm, f$ ) for the semileptonic  $P \rightarrow V\ell\nu_\ell$  decays depending on the current matrix element  $\langle J_{V-A}^\mu \rangle_h$  and the vector meson vertex  $\Gamma^\mu = \gamma^\mu - (P_2 - 2k)^\mu/D$  with various  $D$  factors. Since our findings on the existence/absence of the zero mode are based on the method of power counting, our conclusion applies to other methods of regularization as far as the regularization doesn't change the power counting in the form factor calculation. For example, as discussed by Jaus in Ref. [10], some other multipole type ansatz in the method of regularization wouldn't change the conclusion drawn by the monopole type ansatz.

Table 1: The existence (O(source element)) or absence (X) of the zero-mode contribution to the weak form factors for the semileptonic  $P \rightarrow V\ell\nu_\ell$  decays depending on the current matrix element  $\langle J_{V-A}^\mu \rangle_h$  and the vector meson vertex  $\Gamma^\mu = \gamma^\mu - (P_2 - 2k)^\mu/D$  with various  $D$  factors such as  $D_{\text{con}} \sim (1/x)^0$ ,  $D_{\text{cov}} \sim (1/x)^1$ , and  $D_{\text{LF}} \sim (1/x)^{1/2}$  as  $x \rightarrow 0$ .

|   | $g$                       | $a_+$                     | $a_-$   | $f$  |
|---|---------------------------|---------------------------|---|--|
|   | $\langle J_V^+ \rangle_1$ | $\langle J_A^+ \rangle_1$ | $(\langle J_A^+ \rangle_0, \langle J_A^+ \rangle_1, \langle J_A^\perp \rangle_1)$       | $(\langle J_A^+ \rangle_0, \langle J_A^+ \rangle_1)$ |
| $\gamma^\mu$                            | X                         | X                         | O( $\langle J_A^\perp \rangle_1^{\text{Z.M.}}$ )  | X  |
| $\frac{(P_2 - 2k)^\mu}{D_{\text{con}}}$ | X                         | X                         | O( $\langle J_A^+ \rangle_0^{\text{Z.M.}}, \langle J_A^\perp \rangle_1^{\text{Z.M.}}$ ) | O( $\langle J_A^+ \rangle_0^{\text{Z.M.}}$ )         |
| $\frac{D_{\text{cov}}}{(P_2 - 2k)^\mu}$ | X                         | X                         | X   | X  |
| $\frac{D_{\text{LF}}}{(P_2 - 2k)^\mu}$  | X                         | X                         | X   | X  |

We should note, however, that Jaus's prescription [10] is valid only for the case of  $D = D_{\text{con}}$  but not for the more realistic  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$  case. Essentially, Jaus's prescription corresponds to the replacement  $p_1^-/D \rightarrow -Z_2/D$  for the vector term and  $p_1^- \mathbf{p}_{1\perp}/D \rightarrow -(\mathbf{q}_\perp/D)[A_2^{(1)}Z_2 + (q \cdot P/q^2)A_1^{(2)}]$  for the tensor term regardless of the  $D$  factor used [21]. Indeed he applied this prescription to the more realistic  $D = D_{\text{LF}}$  factor case [10]. However, we show that such prescription is valid only for  $D = D_{\text{con}}$  but not for  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$ . For  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$ , the valid replacement should be  $p_1^-/D \rightarrow 0$  and  $p_1^- \mathbf{p}_{1\perp}/D \rightarrow 0$ .

In terms of current matrix elements,  $\langle J_A^+ \rangle_0^{\text{Z.M.}}$  comes only from  $p_1^-/D$  term but  $\langle J_A^\perp \rangle_0^{\text{Z.M.}}$  comes from both  $p_1^-$  and  $p_1^- \mathbf{p}_{1\perp}/D$  terms. We thus stress that  $\langle J_A^+ \rangle_0^{\text{Z.M.}}$  is absent and the form factor  $f(q^2)$  is immune to the zero mode when  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$  is used. Although the form factor  $a_-(q^2)$  receives the zero-mode contribution from  $\langle J_A^\perp \rangle_0^{\text{Z.M.}}$ , it comes only from the intrinsic  $\gamma^\mu$  part (i.e.  $p_1^- \rightarrow -Z_2$ ) but not from the  $(P_2 - 2k)^\mu/D$  part (i.e.  $p_1^- \mathbf{p}_{1\perp}/D \rightarrow 0$  for  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$ ). Such absence of the zero mode, i.e.  $p_1^-/D \rightarrow 0$  and  $p_1^- \mathbf{p}_{1\perp}/D \rightarrow 0$  are not realized in Jaus's approach [10] for  $D = D_{\text{cov}}$  or  $D_{\text{LF}}$ .

For an explicit demonstration of our findings, we performed the numerical calculation of the  $B \rightarrow \rho$  transition form factors using the model parameters for  $B$  and  $\rho$  used in Refs. [14, 15]. The frame-independence of our results was also checked by comparing the results from the  $q^+ = 0$  frame with those from the  $q^+ > 0$  frame which is summarized in the appendix B. Through the manifestly covariant calculation with  $D = D_{\text{con}}$  and  $D_{\text{cov}}$ , we indeed confirmed that our findings of the zero-mode contributions are correct.

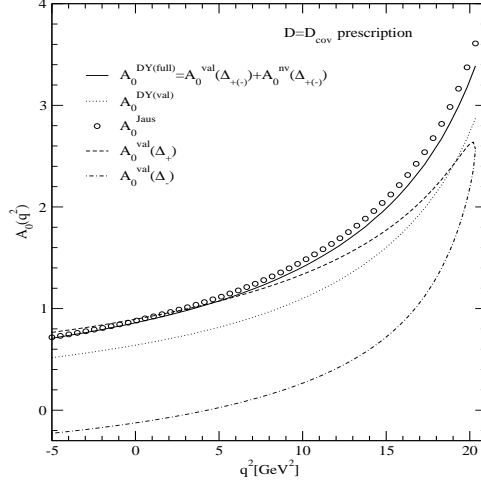


Figure 2: The weak form factor  $A_0(q^2)$  for  $B \rightarrow \rho$  transition for the vector meson vertex  $\Gamma^\mu = \gamma^\mu - (P_2 - 2k)^\mu/D_{\text{cov}}$ .

In Fig. 2, we present the form factor  $A_0(q^2)$  for the vector meson vertex  $\Gamma^\mu = \gamma^\mu - (P_2 - 2k)^\mu/D_{\text{cov}}$ . The solid ( $A_0^{\text{DY(full)}}$ ) and dotted ( $A_0^{\text{DY(val)}}$ ) lines represent the full (i.e. valence + zero-mode) result and the valence contribution in the  $q^+ = 0$  frame, respectively. That is, the difference between the two results (i.e.  $A_0^{\text{DY(full)}} - A_0^{\text{DY(val)}}$ ) represents the zero-mode contribution  $A_0^{\text{Z.M.}}$  to  $A_0(q^2)$ . The circle ( $A_0^{\text{Jaus}}$ ) represents the result obtained from the Jaus's prescription [10]. The dashed and dot-dashed lines represent the valence results obtained from the purely longitudinal  $q^+ > 0$  frame with  $\Delta_+$  and  $\Delta_-$  given by Eq. (B.10), respectively. We note that the valence contribution in the  $q^+ > 0$  frame depends on the direction of the daughter meson recoiling in the positive ( $\Delta_+$ ) or the negative ( $\Delta_-$ )  $z$ -direction relative to the parent meson. Including the nonvalence contribution, however, the full result in the  $q^+ > 0$  frame is in complete agreement with  $A_0^{\text{DY(full)}}$  in the  $q^+ = 0$  frame. On the other hand,  $A_0^{\text{Jaus}}$  shows a small but clear deviation from our full result. Since there is no zero-mode contribution from the  $p_1^-/D_{\text{cov}}$  term, the zero-mode contribution included in the  $D_{\text{con}}$  case is absent in the  $D_{\text{cov}}$  case. Such absence of the zero mode is not realized in Jaus's approach [10]. The numerical deviation between our result (solid line) and Jaus's result (circle) shown in Fig. 2 is due to this difference.

For the  $D = D_{\text{LF}}$  case, although we do not know how to compute the

nonvalence diagram, we can still use our counting rule for the longitudinal momentum fraction factors to check the existence of the zero mode. As summarized in Table 1, the zero-mode contributions from  $p_1^-/D_{\text{LF}}$  and  $p_1^- \mathbf{p}_{1\perp}/D_{\text{LF}}$  do not exist as in the case of  $D_{\text{cov}}$ .

#### 4. Summary and Discussion

In this work, we have analyzed the zero-mode contribution to the weak transition form factors between pseudoscalar and vector mesons. For the phenomenologically accessible vector meson vertex  $\Gamma^\mu = \gamma^\mu - (P_2 - k)^\mu/D$ , we discussed the three typical cases of the  $D$  factor which may be classified by the differences in the power counting of the LF energy (or longitudinal momentum fraction  $x$ )  $p_1^- \sim 1/x$ , i.e.: (1)  $D_{\text{con}} = M_V + m_2 + m \sim (1/x)^0$ , (2)  $D_{\text{cov}} = [2k \cdot P_2 + M_2(m_2 + m) - i\epsilon]/M_2 \sim (1/x)^1$ , and (3)  $D_{\text{LF}} = M'_0 + m_2 + m \sim (1/x)^{1/2}$ . Our main idea to obtain the weak transition form factors is first to find if the zero-mode contribution exists or not for the given form factor using the power counting method. If it exists, then the separation of the on-mass-shell propagating part from the off-mass-shell instantaneous part is useful since the latter is responsible for the zero-mode contribution.

Our findings on the existence/absence of the zero-mode contribution to the weak transition form factors ( $g, a_\pm, f$ ) are summarized in Table 1. We found that the form factors  $g(q^2)$  and  $a_+(q^2)$  are immune to the zero-mode contribution in all three cases of the  $D$  factors. However, the existence/absence of the zero mode in the form factors  $a_-(q^2)$  and  $f(q^2)$  depends on the nature of the  $D$  factors. For the form factor  $f(q^2)$ , while the zero-mode contribution exists in the  $D_{\text{con}}$  case, the other two cases such as  $D_{\text{cov}}$  and  $D_{\text{LF}}$  are immune to the zero-mode contribution. We also should note that the zero-mode contribution to  $f(q^2)$  does not exist in the simple vector meson vertex  $\Gamma^\mu = \gamma^\mu$  (i.e.  $1/D = 0$  case) as we have already shown in [14]. For the form factor  $a_-(q^2)$ , however, the zero-mode contribution exists in the case of  $1/D = 0$ . Including the  $D$  factor, the zero-mode contribution coming from the  $D$  factor exists in the  $D_{\text{con}}$  case but not in the other two cases of  $D_{\text{cov}}$  and  $D_{\text{LF}}$ . That is, if one uses the  $D_{\text{cov}}$  or  $D_{\text{LF}}$  in the phenomenological vector meson vertex, our results show that the form factors  $g(q^2)$ ,  $a_+(q^2)$ , and  $f(q^2)$  are immune to the zero-mode contribution, but the form factor  $a_-(q^2)$  receives the zero mode coming only from the simple vertex  $\gamma^\mu$  term. We also found the corresponding zero-mode operator for  $a_-(q^2)$  that is convoluted with the initial and final state LF wave functions (see Eq. (27)). This pro-

vides a well-established basis of LF approach to compute the weak transition form factors between pseudoscalar and vector mesons without missing any zero-mode contribution. The covariance (i.e., frame independence) of our model for the cases of  $D = D_{\text{con}}$  and  $D_{\text{cov}}$  has been checked by performing the LF calculation in the  $q^+ = 0$  frame in parallel with the purely longitudinal  $q^+ > 0$  frame using the exactly solvable covariant fermion field theory model in  $(3 + 1)$  dimensions.

All of these findings stem from the fact that the zero-mode contribution from the  $D$  factor is absent if the denominator  $D$  of the vector meson vertex  $\Gamma^\mu = \gamma^\mu - (P_2 - k)^\mu/D$  contains the term proportional to the LF energy (or longitudinal momentum fraction  $x$ )  $(p_1^-)^n \sim (1/x)^n$  with the power  $n > 0$ . While the correct implementation of zero-mode contributions cannot solve all the problems in the phenomenology, at least the Lorentz covariance of the result can be assured in the LFQM. This certainly benefits the hadron phenomenology.

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## Appendix A. Trace terms $(S_V^\mu)_h$ and $(S_A^\mu)_h$ in Eq. (6)

In this appendix we summarize the LF results of the trace terms  $[S_{V(A)}^\mu]_h$  in Eq. (6) by separating it into the on-mass-shell propagating part  $[S_{V(A)}^\mu]_h^{\text{on}}$  and the off-mass-shell instantaneous part  $[S_{V(A)}^\mu]_h^{\text{inst}}$  as follows

$$\begin{aligned} (S_V^\mu)_h^{\text{on}} = & 4i\varepsilon^{\mu\nu\rho\sigma} \left\{ [m_1 p_{2\text{on}\nu} k_{\text{on}\rho} - m_2 p_{1\text{on}\nu} k_{\text{on}\rho} - m p_{2\text{on}\nu} p_{2\text{on}\rho}] \epsilon_\sigma^* \right. \\ & \left. + \frac{2k_{\text{on}} \cdot \epsilon^*}{D_{\text{on}}} p_{1\text{on}\nu} p_{2\text{on}\rho} k_{\text{on}\sigma} \right\}, \end{aligned} \quad (\text{A.1})$$



$$\begin{aligned}
(S_V^\mu)_h^{\text{inst}} &= 2i\varepsilon^{\mu\nu+\sigma} \left\{ [\delta p_1^-(mp_{2\text{on}} + m_2k_{\text{on}})_\nu - \delta p_2^-(mp_{1\text{on}} + m_1k_{\text{on}})_\nu \right. \\
&\quad \left. - \delta k^-(m_2p_{1\text{on}} - m_1p_{2\text{on}})_\nu] \epsilon_\sigma^* - \frac{2k_{\text{on}} \cdot \epsilon^* + \epsilon^{*+} \delta k^-}{D_{\text{on}} + \delta D} \right. \\
&\quad \left. \times [p_{2\text{on}\nu} k_{\text{on}\sigma} \delta p_1^- - p_{1\text{on}\nu} k_{\text{on}\sigma} \delta p_2^- + p_{1\text{on}\nu} p_{2\text{on}\sigma} \delta k^-] \right\} \\
&\quad + 4i\varepsilon^{\mu\nu\rho\sigma} \frac{\delta k^-}{D_{\text{on}} + \delta D} \epsilon^{*+} p_{1\text{on}\nu} p_{2\text{on}\rho} k_{\text{on}\sigma}, \tag{A.2}
\end{aligned}$$

and

$$\begin{aligned}
(S_A^\mu)_h^{\text{on}} &= 4m_1[(k_{\text{on}} \cdot \epsilon^*)p_{2\text{on}}^\mu + (p_{2\text{on}} \cdot \epsilon^*)k_{\text{on}}^\mu - (k_{\text{on}} \cdot p_{2\text{on}})\epsilon^{*\mu}] \\
&\quad - 4m_2[(k_{\text{on}} \cdot \epsilon^*)p_{1\text{on}}^\mu + (p_{1\text{on}} \cdot \epsilon^*)k_{\text{on}}^\mu + (k_{\text{on}} \cdot p_{1\text{on}})\epsilon^{*\mu}] \\
&\quad + 4m[(p_{2\text{on}} \cdot \epsilon^*)p_{1\text{on}}^\mu + (p_{1\text{on}} \cdot \epsilon^*)p_{2\text{on}}^\mu - (p_{1\text{on}} \cdot p_{2\text{on}})\epsilon^{*\mu}] \\
&\quad + 4mm_1m_2\epsilon^{*\mu} - 8\frac{k_{\text{on}} \cdot \epsilon^*}{D_{\text{on}}}[(p_{2\text{on}} \cdot k_{\text{on}} - m_2m)p_{1\text{on}}^\mu \\
&\quad + (p_{1\text{on}} \cdot k_{\text{on}} + m_1m)p_{2\text{on}}^\mu - (p_{1\text{on}} \cdot p_{2\text{on}} + m_1m_2)k_{\text{on}}^\mu], \tag{A.3}
\end{aligned}$$

$$\begin{aligned}
(S_A^\mu)_h^{\text{inst}} = & -2\delta k^- \left\{ m_1[p_2^+ \epsilon^{*\mu} - p_{2\text{on}}^\mu \epsilon^{*+} - (p_{2\text{on}} \cdot \epsilon^*) g^{\mu+}] + m_2[p_1^+ \epsilon^{*\mu} \right. \\
& + p_{1\text{on}}^\mu \epsilon^{*+} - (p_{1\text{on}} \cdot \epsilon^*) g^{\mu+}] + \frac{2k_{\text{on}} \cdot \epsilon^* + \epsilon^{*+} \delta k^-}{D_{\text{on}} + \delta D} [p_1^+ p_{2\text{on}}^\mu \\
& + p_{1\text{on}}^\mu p_2^+ - (p_{1\text{on}} \cdot p_{2\text{on}} + m_1 m_2) g^{\mu+}] \left. \right\} \\
& -2\delta p_1^- \left\{ m_2[k^+ \epsilon^{*\mu} - k_{\text{on}}^\mu \epsilon^{*+} + (k_{\text{on}} \cdot \epsilon^*) g^{\mu+}] + m[p_2^+ \epsilon^{*\mu} \right. \\
& - p_{2\text{on}}^\mu \epsilon^{*+} - (p_{2\text{on}} \cdot \epsilon^*) g^{\mu+}] + \frac{2k_{\text{on}} \cdot \epsilon^* + \epsilon^{*+} \delta k^-}{D_{\text{on}} + \delta D} [p_{2\text{on}}^\mu k^+ \\
& - p_2^+ k_{\text{on}}^\mu + (p_{2\text{on}} \cdot k_{\text{on}} - m_2 m) g^{\mu+}] \left. \right\} \\
& -2\delta p_2^- \left\{ m_1[k^+ \epsilon^{*\mu} - k_{\text{on}}^\mu \epsilon^{*+} - (k_{\text{on}} \cdot \epsilon^*) g^{\mu+}] + m[p_1^+ \epsilon^{*\mu} \right. \\
& - p_{1\text{on}}^\mu \epsilon^{*+} - (p_{1\text{on}} \cdot \epsilon^*) g^{\mu+}] + \frac{2k_{\text{on}} \cdot \epsilon^* + \epsilon^{*+} \delta k^-}{D_{\text{on}} + \delta D} [p_{1\text{on}}^\mu k^+ \\
& - p_1^+ k_{\text{on}}^\mu + (p_{1\text{on}} \cdot k_{\text{on}} - m_1 m) g^{\mu+}] \left. \right\} + 2m_1 \epsilon^{*+} g^{\mu+} \delta p_2^- \delta k^- \\
& + 2g^{\mu+} \delta p_1^- \delta p_2^- \left\{ m \epsilon^{*+} - \frac{2k_{\text{on}} \cdot \epsilon^* + \epsilon^{*+} \delta k^-}{D_{\text{on}} + \delta D} k^+ \right\} \\
& - 4 \frac{\delta k^-}{D_{\text{on}} + \delta D} \epsilon^{*+} [(p_{2\text{on}} \cdot k_{\text{on}} - m_2 m) p_{1\text{on}}^\mu \\
& + (p_{1\text{on}} \cdot k_{\text{on}} + m_1 m) p_{2\text{on}}^\mu - (p_{1\text{on}} \cdot p_{2\text{on}} + m_1 m_2) k_{\text{on}}^\mu], \quad (\text{A.4})
\end{aligned}$$

where  $D_{\text{on}}$  is the denominator factor  $D$  when  $k = k_{\text{on}}$  and  $\delta D$  is the difference between  $D$  and  $D_{\text{on}}$ , i.e.  $\delta D = D(k) - D_{\text{on}}(k_{\text{on}})$ . For the  $D_{\text{con(LF)}}$  factor,  $\delta D_{\text{con(LF)}} = 0$  and  $D_{\text{on}} = D_{\text{con(LF)}}$ . For the  $D_{\text{cov}}$  factor including the four momentum  $k$  explicitly, however, one obtains  $\delta D_{\text{cov}} = \delta k^- (P_2^+ / M_2)$ . In Eq. (A.4), we have also used  $P_2 \cdot \epsilon^* = 0$ .

## Appendix B. Form factors in the purely longitudinal frame

The purpose of this appendix is to show the frame-independence (i.e. covariance) of our result obtained from the  $q^+ = 0$  frame by comparing with the result obtained from the  $q^+ > 0$  frame, which is summarized in this appendix.

In the reference frame where  $q^+ > 0$  and  $\mathbf{P}_{1\perp} = 0$ , the (timelike) momentum transfer  $q^2 = (P_1 - P_2)^2$  is given by

$$q^2 = q^+ q^- - \mathbf{q}_\perp^2 = \Delta \left( M_1^2 - \frac{M_2^2}{1 - \Delta} \right) - \frac{\mathbf{q}_\perp^2}{1 - \Delta}, \quad (\text{B.1})$$

where  $q^+ = \Delta P_1^+$ . In this frame, only the plus component of the  $V-A$  current can be utilized for the calculations of LF valence[Fig. 1(b)] and nonvalence [Fig. 1(c)] diagrams.

In the valence region  $0 < k^+ < P_2^+$  (i.e.  $\Delta < x < 1$ ), the pole at  $k^- = k_{\text{on}}^-$  (from the spectator quark) is located in the lower half plane of the complex  $k^-$  variable. Thus, the Cauchy integration formula for the  $k^-$ -integral in Eq. (4) yields

$$\langle J_h^\mu \rangle^{\text{val}} = \frac{N}{16\pi^3} \int_\Delta^1 \frac{dx}{1-x} \int d^2\mathbf{k}_\perp \chi_1(x, \mathbf{k}_\perp) S_h^\mu(k_{\text{on}}^-) \chi_2(x', \mathbf{k}'_\perp), \quad (\text{B.2})$$

where  $x' = (x - \Delta)/(1 - \Delta)$  and  $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1 - x')\mathbf{q}_\perp$ .

In the nonvalence region  $P_2^+ < k^+ < P_1^+$  (i.e.  $0 < x < \Delta$ ) the poles at  $p_1^- = p_{1\text{on}}^-(m_1)$  (from the struck quark propagator) and  $p_1^- = p_{1\text{on}}^-(\Lambda_1)$  (from the smeared quark-gauge-boson vertex) are located in the upper half plane of the complex  $k^-$  variable. Thus, the Cauchy integration over  $k^-$  in Eq. (4) yields

$$\begin{aligned} \langle J_h^\mu \rangle^{\text{nv}} &= \frac{N}{16\pi^3(\Lambda_1^2 - m_1^2)} \int_0^\Delta \frac{dx}{(1-x)(\Delta-x)x''(1-x'')} \\ &\times \int d^2\mathbf{k}_\perp \left\{ \frac{S_h^\mu(p_{1\text{on}}^-(\Lambda_1))}{(M_1^2 - M_{\Lambda_1}^2)(q^2 - M_{\Lambda_1\Lambda_2}^2)(q^2 - M_{\Lambda_1 m_2}^2)} \right. \\ &\quad \left. - \frac{S_h^\mu(p_{1\text{on}}^-(m_1))}{(M_1^2 - M_0^2)(q^2 - M_{m_1\Lambda_2}^2)(q^2 - M_{m_1 m_2}^2)} \right\}, \quad (\text{B.3}) \end{aligned}$$

where

$$M_{ab}^2 = \frac{\mathbf{k}_\perp'^2 + a^2}{x''} + \frac{\mathbf{k}_\perp''^2 + b^2}{1 - x''}, \quad (\text{B.4})$$

and

$$x'' = \frac{x}{\Delta}, \quad \mathbf{k}_\perp'' = \mathbf{k}_\perp + x''\mathbf{q}_\perp. \quad (\text{B.5})$$

The explicit forms of the trace terms  $(S_{V-A}^+)^{\text{nv}}_h(p_{\text{1on}}^-(\Lambda_1))$  for the vector and axial-vector currents are given by

$$\begin{aligned}
(S_V^+)^{\text{nv}}_{h=1} &= -\frac{2}{\sqrt{2}}\varepsilon^{+-xy}\left\{q^L\mathcal{A}_1 + k^L[m_1 - m_2 - \Delta(m_1 - m)]\right. \\
&\quad \left. + \frac{2}{D}[\mathbf{k}_\perp^2 q^L - (\mathbf{k}_\perp \cdot \mathbf{q}_\perp)k^L]\right\}, \\
(S_A^+)^{\text{nv}}_{h=1} &= \frac{4}{\sqrt{2}}\left\{(2x' - 1)q^L\mathcal{A}_1 + k^L[(2x - 1 - \Delta)(m_1 - m) - m_2 - m]\right. \\
&\quad \left. - \frac{2((1 - x')q^L + k^L)}{(1 - x')D}[\mathbf{k}_\perp \cdot \mathbf{k}'_\perp + \mathcal{A}_1\mathcal{B}'_2\right. \\
&\quad \left. + xx'(1 - x)(M_1^2 - M_{\Lambda_1}^2)]\right\}, \\
(S_A^+)^{\text{nv}}_{h=0} &= -\frac{4}{(1 - x')M_2}\left\{\mathcal{A}_1[x'(1 - x')M_2^2 + m_2m - (1 - x')^2q^2]\right. \\
&\quad + \mathbf{k}_\perp^2(\mathcal{A}_1 + m_2 - m) + (1 - x')\mathbf{k}_\perp \cdot \mathbf{q}_\perp(2\mathcal{A}_1 + m_2 - m) \\
&\quad + x(1 - x)m_2(M_1^2 - M_{\Lambda_1}^2) - \frac{1}{xD}[x(1 - x')M_2^2 - (1 - \Delta)xM_1^2 \\
&\quad + (1 - \Delta)(\Lambda_1^2 + \mathbf{k}_\perp^2) + x(1 - x')q^2 - 2x\mathbf{k}_\perp \cdot \mathbf{q}_\perp][\mathbf{k}_\perp \cdot \mathbf{k}'_\perp + \mathcal{A}_1\mathcal{B}'_2 \\
&\quad \left. + xx'(1 - x)(M_1^2 - M_{\Lambda_1}^2)]\right\}, \tag{B.6}
\end{aligned}$$

where  $\mathcal{A}_i = (1 - x)m_i + xm$  ( $i = 1, 2$ ) and  $\mathcal{B}'_2 = x'm - (1 - x')m_2$ . The trace terms  $S_h^+(p_{\text{1on}}^-(m_1))$  for the vector and axial-vector currents can be obtained by the replacement  $\Lambda_1 \rightarrow m_1$  in Eq. (B.6). We also note that the trace terms  $S_h^+(p_{\text{1on}}^-(\Lambda_1))$  and  $S_h^+(p_{\text{1on}}^-(m_1))$  in the nonvalence region include both the on-mass-shell quark propagating part and the off-mass-shell instantaneous part.

The relations between the current matrix elements and the weak form factors in this  $q^+ > 0$  frame are as follows:

$$\langle J_V^+ \rangle_{h=1} = -\frac{1}{\sqrt{2}}\varepsilon^{+-xy}q^L g(q^2), \tag{B.7}$$

for the vector current and

$$\begin{aligned}\langle J_A^+ \rangle_{h=1} &= \frac{q^L}{(1-\Delta)\sqrt{2}} \left[ (2-\Delta)a_+(q^2) + \Delta a_-(q^2) \right], \\ \langle J_A^+ \rangle_{h=0} &= \frac{1-\Delta}{2M_2} \left\{ 2f(q^2) + \left[ M_1^2 - \frac{M_2^2}{(1-\Delta)^2} \right. \right. \\ &\quad \left. \left. - \frac{q^2}{(1-\Delta)^2} \right] [(2-\Delta)a_+(q^2) + \Delta a_-(q^2)] \right\},\end{aligned}\quad (\text{B.8})$$

for the axial-vector current. In the purely longitudinal momentum  $q^+ > 0$  and  $\mathbf{q}_\perp = 0$  frame, where

$$q^2 = q^+ q^- = \Delta \left( M_1^2 - \frac{M_2^2}{1-\Delta} \right), \quad (\text{B.9})$$

there are two solutions of  $\Delta$  for a given  $q^2$ , i.e.,

$$\Delta_\pm = \frac{M_1^2 - M_2^2 + q^2 \mp \sqrt{(M_1^2 - M_2^2 + q^2)^2 - 4M_1^2 q^2}}{2M_1^2}, \quad (\text{B.10})$$

where the  $+$ ( $-$ ) sign in Eq. (B.10) corresponds to the daughter meson recoiling in the positive(negative)  $z$ -direction relative to the parent meson.

At zero recoil ( $q^2 = q_{\text{max}}^2$ ) and maximum recoil ( $q^2 = 0$ ),  $\Delta_\pm$  are given by

$$\begin{aligned}\Delta_+(q_{\text{max}}^2) &= \Delta_-(q_{\text{max}}^2) = 1 - \frac{M_2}{M_1}, \\ \Delta_+(0) &= 0, \quad \Delta_-(0) = 1 - \left( \frac{M_2}{M_1} \right)^2.\end{aligned}\quad (\text{B.11})$$

The form factors should in principle be independent of the recoil directions ( $\Delta_\pm$ ) if the nonvalence contributions are added to the valence ones. While the form factor  $g(q^2)$  in the  $q^+ > 0$  frame can be obtained directly from Eq. (10), the form factor  $f(q^2)$  can be obtained only after  $a_\pm(q^2)$  are calculated.

To illustrate this, we define

$$\langle J_A^+ \rangle_{h=1} |_{\Delta=\Delta_\pm} \equiv \frac{q^L}{\sqrt{2}} I_A^+(\Delta_\pm). \quad (\text{B.12})$$

Then we obtain from Eq. (B.8)

$$\begin{aligned}
a_+(q^2) &= \frac{1}{2(\Delta_- - \Delta_+)} [\Delta_- (1 - \Delta_+) I_A^+(\Delta_+) - \Delta_+ (1 - \Delta_-) I_A^+(\Delta_-)], \\
a_-(q^2) &= \frac{1}{2(\Delta_+ - \Delta_-)} [(1 + \Delta_+) (2 - \Delta_-) I_A^+(\Delta_+) \\
&\quad - (1 - \Delta_-) (2 - \Delta_+) I_A^+(\Delta_-)],
\end{aligned} \tag{B.13}$$

and

$$f(q^2) = \frac{M_2}{1 - \Delta} \langle J_A^+ \rangle_{h=0} - \frac{1}{2} \left( M_1^2 - \frac{M_2^2}{(1 - \Delta)^2} \right) \left[ (2 - \Delta) a_+(q^2) + \Delta a_-(q^2) \right], \tag{B.14}$$

where  $\Delta$  in Eq. (B.14) can be either  $\Delta_+$  or  $\Delta_-$ . As one can see from Eqs. (B.7) and (B.8), one should be careful in setting  $\mathbf{q}_\perp = 0$  to get the correct results in the purely longitudinal frame. One cannot simply set  $\mathbf{q}_\perp = 0$  from the start, but may set it to zero only after the form factors are extracted.

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